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Development of Interleaver for the BICM-ID system based on the GBIm

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Abstract– **In this study, the Generalized Block Interleaving method (GBIm) is proposed for the Bit-Interleaved Coded Modulation with Iterative Decoding (BICM-ID) system. Generalized Block Interleaving with Almost Regular Permutation (GBI-ARP) and Generalized Block Interleaving with Golden (GBI-Golden) are developed based on GBIm, utilizing component interleaving constructed from algebraic mathematical expressions. With this approach, the proposed interleavers maintain simplicity while ensuring high randomness and large connectivity indices. This enhances system flexibility and reduces memory requirements. Research results demonstrate that the GBI-ARP and GBI-Golden interleavers significantly improve the Bit Error Rate (BER) performance of the BICM-ID system using (8,4,4) Extended Hamming code and 16-QAM modulation, achieving a gain of 0.5 dB in Signal-to-Noise Ratio (SNR) at a BER of 10^{-6} compared to a random interleaver, and over 2dB at a BER of 10^{-4} compared to traditional Block interleaver and basic Golden interleaver. Furthermore, the proposed interleavers based on the GBIm meet the criteria for complexity, latency, and applicability in next-generation real-time communication systems.**

Keywords– **BICM-ID, BIBCM-ID, generalized block interleaving, interleaver, Hamming codes, channel coding.**

1 Introduction

Coded modulation enhances communication system quality through the optimal combination of channel coding and modulation. This approach has proven effective for next-generation wireless transmission systems. Traditional coded modulation schemes include Block Coded Modulation (BCM) [1] and Trellis Coded Modulation (TCM) [2].

Recently, solutions aimed at improving transmission quality and optimizing bandwidth utilization have emerged, utilizing Bit Interleaved Coded Modulation (BICM) [3, 4], BICM-ID [5–8], as well as advancements such as BIBCM-ID [9], BIPCM-ID [10, 11], and Turbo BICM-ID [12]. The utilization of coded modulation structures in conjunction with iterative decoding has demonstrated significant efficiency improvements in both Additive White Gaussian Noise (AWGN) and fading channels [6, 13, 14].

In the context of BICM-ID systems, the interleaving technique is of paramount importance. Specifically, during the signal processing at the receiver, the interleaver serves a critical function within the iterative decoding framework. The pivotal role of the interleaver becomes particularly pronounced in the iterative processing operations inherent to the BIBCM-ID scheme. An optimally designed interleaver can substantially enhance the convergence rate of soft decoding and iterative decoding algorithms within the BICM-ID architecture, thereby improving decoding accuracy and overall system performance. Consequently, to fully exploit the capabilities of the BICM-ID structure, it is imperative to not only develop efficient encoding/decoding and modulation/demodulation techniques but also to devise a suitable interleaver.

The utilization of interleavers not only enhances the transmission efficiency over Gaussian channels but also serves as a critical solution for improving the reliability and effectiveness of communication over fading channels. This is an issue that many advanced modulation techniques with high robustness, such as those employed in 4G and 5G networks, have yet to fully address. Consequently, interleavers have become an essential component for optimizing transmission performance, thereby meeting the increasingly stringent standards of modern digital transmission systems.

In recent years, the research and design of interleavers for wireless communication systems have been extensively pursued, yielding various solutions for error correction codes such as Turbo codes, LDPC codes, and Polar codes, which allow for flexible code rate adjustments [15–17], low-complexity implementations of interleaver/deinterleaver architectures for WiMAX [18], as well as interleaving techniques utilizing balanced square or trapezoidal interleavers for Polar codes [19]. Furthermore, studies have focused on memory sharing and hardware implementation for block interleaving in 4G Turbo codes and bit interleaving in 5G LDPC codes [20, 21].

Despite these advancements, current proposals predominantly emphasize enhancing code quality or addressing channel interleaving issues without fully exploring the intricate relationship between modulation and coding. This oversight creates a gap in the development of efficient interleaver architectures that can minimize complexity in modern digital transmission systems employing combined modulation-coding structures, such as the Bit Interleaved Coded Modulation with Iterative Decoding (BICM-ID).

For the BICM-ID system Traditional block interleavers [22, 23], algebraic interleavers such as the Almost Regular Permutation (ARP) interleaver [24, 25] and pseudo-random interleavers such as the Golden interleavers [26, 27] can be commonly employed. However, their efficacy is limited within the BICM-ID system framework, particularly concerning the restricted connectivity indices between bits in M-QAM signals and coded bits. In contrast, random interleavers exhibit high connectivity indices between coded bits and signal symbols but are primarily applicable for research purposes and lack practical implementation feasibility. Therefore, to harness the benefits of bit-interleaved coded modulation and iterative decoding in BICM-ID systems, it is essential to propose a systematic approach that enables the construction of interleavers characterized by high randomness and low computational complexity. To address these challenges, this paper presents a methodology for constructing interleavers in BICM-ID systems that aligns with the requirements for complexity, randomness characteristics and applicability in real-time communication systems.

The structure of this paper is organized as follows. Section 2 introduces the BICM-ID system model. Section 3 details the algorithm for constructing interleavers utilizing the GBIm. Section 4 presents simulation results and analyzes the performance quality of the BICM-ID system employing interleavers constructed via the proposed methodology. Finally, Section 5 concludes the paper with a summary of findings and implications for future research.

2 BIBCM-ID System

In this section, we describe the structure of the BICM-ID system, which utilizes block coding, specifically the BIBCM-ID system. The BIBCM-ID model employs block codes of limited length, particularly extended Hamming codes, in conjunction with M-QAM modulation, as illustrated in Figure 1.

At the input of the block encoder, a bit stream is divided into blocks, each containing k bits. Each k-bit data block is encoded by the block encoder $C(n,k,d)$ into a codeword consisting of n bits. The output bit stream $N_{cb} = N_{cw}k$ from the encoder is interleaved by a interleaver π of length N, and then divided into blocks, each containing m bits, satisfying the condition $N_p = N_{cw}k = N_s m$. The interleaver specifies how to group the bits in the binary label of the modulated signal from the bits in the data frame of length N_{cb}. Each group of m bits at the output of the interleaving

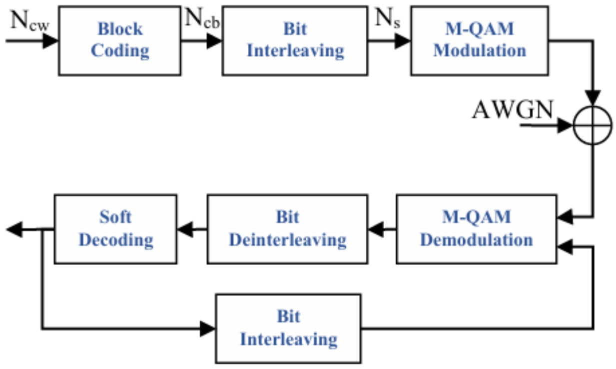

Figure 1. Block-diagram of the BIBCM-ID system.

function π is used as a binary label to map onto the constellation of M-QAM modulation signals $(m = log_2 M)$.

At the receiver, the M-QAM signal, affected by AWGN, is input into the demodulator. Under ideal feedback conditions, each encoded bit is transmitted through one of the m equivalent Symmetric Binary Channels (EBSC). The soft demodulator computes the Log-Likelihood Ratio (LLR) and provides information to the decoder for decoding within an iterative processing structure that integrates demodulation, decoding, interleaving and de-interleaving.

The decoder employs a Soft-Input Soft-Output (SISO) decoding algorithm based on dual decoding methods [28], with the output value represented as a likelihood ratio in logarithmic form, which provides feedback to the soft demodulator via the interleaving function π. Thus, at each iteration, the input to the soft decoder for the Extended Hamming code is derived from the output of the soft demodulator after being de-permuted by π^{-1}. The information at the output of the demodulator, combined with extrinsic information received from the block decoder through the interleaving function π and de-interleaving π^{-1} after a certain number of iterations, is hard-decided to yield the received bit information. Therefore, the role of the interleaving function within this structure is crucial, significantly enhancing the convergence rate of the soft decoding algorithm in the BIBCM-ID scheme, which translates to improved decoding accuracy and overall system performance. In BICM-ID schemes, both quality and complexity of the system heavily depend on the bit interleaver utilized and are directly related to the Bit Error Rate (BER) floor of the system [9].

The modulation process through mapping under conditions where bits in the binary label comprising m bits of the M-QAM modulated signal are not entirely independent allows for leveraging the reliability of bits to enhance the reliability of others. The modulation process through mapping under the condition, where the bits in the m-bit binary label of the M-QAM modulated signal are not entirely independent, allows the reliability of certain bits to enhance the reliability of others. Furthermore, the fact that each bit in the binary label of the signal can belong to a codeword different from those containing the remaining bits enables the use of post-decoding information to improve the reliability of all bits in the signal's binary label. This process can be represented using a Bipartite graph, as illustrated in Figure 2.

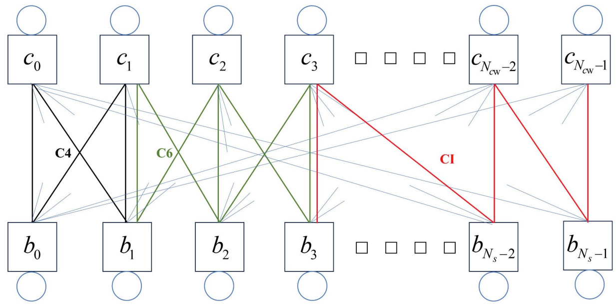

Figure 2. Relations between code and signal nodes of a Bipartite Graph.

In Figure 2, the code nodes are subsets of the indices of the bits in a codeword within a data frame of length N_{cb} bits and are denoted as c_i, $0 \leq i \leq N_{cw} - 1$. The nodes below are signal nodes, denoted as $(b_j,\ 0 \leq j \leq N_s 1)$, they are subsets of the indices of the bits in the data frame of length N_{cb} bits, which after interleaving become the binary labels of the same signal. At the receiver side, during the iterative demodulation/decoding process, each signal node b_j provides post-demodulation information to mn code nodes c_i, $(m = \log_2 M)$ which belong to m different codewords of length n bits.

Consequently, the effectiveness of the iterative demodulation/decoding method depends on the information exchanged between the signal nodes and the code nodes. Therefore, the role of the interleaving function within this structure is crucial, significantly enhancing the convergence rate of the soft decoding algorithm. In the subsequent iteration, the m decoded codewords feed information back to m signal nodes $b_{j'}$, including the signal node b_j itself. In this case, the connection from b_j through c_i back to b_j forms a self-loop, and the information within the self-loop does not contribute to the demodulation of other signals via decoding. The relationship between the code nodes c_i and the signal nodes b_j, which are determined by the interleaver π, can be represented using a relationship matrix D as in Algorithm 1 [9].

Based on the Bipartite graph (Figure 2), it can be observed that the relationship between code nodes and signal nodes is both reflexive and symmetric. Each signal node has a self-loop, representing the processes of demodulation (for signals) and decoding (for codewords). A short self-loop, referred to as a "4-step error loop," occurs when there is an intersection of two pairs of codewords and two pairs of signal labels, characterized by the connection index $C4$ (4-Cycles). Similarly, if three codewords intersect with the binary labels of three signals in such a way that each codeword contributes 2 bits to the binary labels of two signals, it results in a "6-step error loop".

Algorithm 1 : Determining the relationship matrix
Given permutation π and $0 \leq k \leq n-1, 0 \leq l \leq m-1$ setting up the matrix D

1) Set up the matrix P:

$$p_{in+k,jm+l} = \begin{cases} 1 & \text{if } \pi(in+k) = (jm+l) \\ 0 & \text{if } \pi(in+k) \neq (jm+l) \end{cases}$$

2) Divide P into sub-matrices

$$P(i,j) = \{p_{in+k,jm+l}\}$$

3) Compute $s(i,j)$

$$s(i,j) = \sum_{k=0}^{n-1} \sum_{l=0}^{m-1} p_{in+k,jm+l}$$

4) Determine

$$D = \{d_{i,j}, 0 \leq i \leq N_{cw}-1, 0 \leq j \leq N_s - 1\}$$

$$d_{i,j} = \begin{cases} 1 & \text{if } s(i,j) > 0 \\ 0 & \text{if } s(i,j) = 0 \end{cases}$$

D is a regular matrix if and only if $0 \leq s(i,j) \leq 1$ for all $0 \leq i \leq N_{cw} - 1$ and $0 \leq j \leq N_s - 1$.

The number of short closed loops such as of "4t-Cycles" is

$$C(4t) = \frac{1}{4t} \left(\sum_{j_1=0}^{N_s-1} \sum_{j_2=0}^{N_s-1} E_{j_1,j_2}^{(t)} \left(E_{j_1,j_2}^{(t)} - 1 \right) \right), \quad (1)$$

and the number of "6-Cycles", "10-step" ..., denoted as $C(4t+2)$

$$C(4t+2) = \frac{1}{4t+2} \left(\sum_{j_1=0}^{N_s-1} \sum_{j_2=0}^{N_s-1} E_{j_1,j_2}^{(2t)} E_{j_1,j_2}^{(1)} \right), \quad (2)$$

where, E_{j_1,j_2}, $(0 \leq j_1 \neq j_2 \leq N_s - 1)$ is the number of paths connecting the signal node b_{j_1} through the code nodes c_i back to the signal node b_{j_2}, $E^{(1)} = DD^T$, D^T is the transpose of the relationship matrix D.

When considering a set of signals consisting of N_s signal nodes, we focus on the Connection Index (CI) of the bipartite graph. The Connection Index, denoted CI, to the average value over all N_s signal nodes b_j và and is defined as

$$CI = \frac{1}{N_s} \left(\sum_{j_1=0}^{N_s-1} \sum_{j_2=0}^{N_s-1} e_{j_1,j_2} \right), \quad (3)$$

where, e_{j_1,j_2} lis the total number of paths connecting the signal node b_{j_1} to b_{j_2} through the Bipartite graph.

An increased index CI indicates a higher number of connections between code nodes through signal nodes. This enhancement contributes to improved reliability during the decoding process. Correctly decoding a codeword not only conveys information about that specific codeword but also provides valuable information regarding other codewords throughout the iterative decoding process. As the number of connections increases, the amount of extrinsic information or reliability that each decoding iteration delivers is also amplified. The CI is determined by the *Algorithm* 2 [9].

Algorithm 2 : Computing Connection Index

1) Compute $E = DD^T$
2) Compute $E^{(2)} = E^{(1)} \cdot E^{(1)}$, $E^{(3)}, \ldots$
3) Compute $C(4t), C(4t+2)$ for $t \geq 1$
4) Compute CI

Where $E^{(2)} = E^{(1-)} \cdot E^{(1-)}$, $E^{(2)}_{i_1,i_2}$ is the number of connections from a node c_i, $i \neq i_1$, through a node b_j, then from c_i through a node $b_{j'} \neq b_j$ to the node $c_{i_2}, i_2 \neq i_1$.

During the exchange of extrinsic information between demodulation and decoding, the interleaving plays a crucial role. When the number of links between a signal node and other signal nodes through the code node is larger, the efficiency of decoding/demodulation is higher. Therefore, the criteria for evaluating a good interleaving block are based on the connection metrics $C4, C6, C8$ and CI. A good interleaving block is one that has $CI \approx N_p$ and $C4 \approx 0$ [9]. A good block interleaving has a high CI, which corresponds to the property that a code bit has many connections with symbols or a symbol has many connections with codewords.

3 Algorithm for Constructing Interleaver Using the GBIm

As previously mentioned, the construction of good interleavers plays a very important role in improving the system quality. The design issue of different interleavers with high randomness characteristics, meeting requirements on complexity, BER performance improvement, and applicability, can be addressed based on the GBIm.

The simplest interleaver is the block interleaver, constructed in a "write-by-row, read-by-column" manner. However, for block interleavers, the number of "4-step cycles" and "6-step cycles" is relatively large, causing the phenomenon of "looping" in iterative decoding and leading to an error floor phenomenon during decoding. To overcome this limitation, the General Block Interleaver (GBI) based on the GBIm is constructed by writing by row, reading by column after each column is permuted by component interleaver. Denote π as the GBI with length N_p

$$\begin{aligned}\pi = (&q_{\pi_1(1),1}, q_{\pi_1(2),1}, \cdots, q_{\pi_1(N_{cw}),1},\\ &q_{\pi_2(1),2}, q_{\pi_2(2),2}, \cdots, q_{\pi_2(N_{cw}),2},\\ &\cdots,\\ &q_{\pi_n(1),n}, \cdots, q_{\pi_n(N_{cw}),n}),\end{aligned} \tag{4}$$

where π_k with $1 \leq k \leq n$ is an interleaving on the set of natural numbers $\{1, 2, \cdots, N_{cw}\}$; $q_{ik}, 1 \leq i \leq N_{cw}$ are numbers belonging to $\mathbb{N}$ that have the same remainder k when divided by n. Then, the interleaving π can be defined as follows

$$\pi(i + (k-1)N_{cw}) = (\pi_k(i) - 1)n + k. \tag{5}$$

The GBI permutation algorithm is implemented as follows

Algorithm 3 : Generating the GBI

1) Determine the triple N_p, n, m such that nm divides N_p.
2) Let $M = \frac{N_p}{n}, N = \frac{N_p}{m}$.
3) For $0 \leq k \leq n-1$, generate π_k as the permutation on the set $\{0, 1, \cdots, M-1\}$.
4) For $0 \leq i \leq M-1$, compute

$$\pi(in + k) = \pi_k(i) + kM$$

π is a GBI if and only if $0 \leq s(i,j) \leq 1$ for all i, $0 \leq i \leq M-1$ and $0 \leq j \leq N-1$.

In the general case, the parameters i are generated randomly. However, since the parameters i are generated randomly, the random GBI cannot be accurately reconstructed at the receiver side in practice. In order to find a good permutation set that meets the requirements for the connection indices $C4$, $C6$, $C8$, and CI , the search process (exhaustive search) can be performed according to the following Algorithm 4

Algorithm 4 : Searching the best interleaver

1) Set up GBI π using *Algorithm* 3
2) Set up the matrix D associated with π using *Algorithm* 1
3) If D is full rank, compute $C_4(t)$, $C_4(t+2)$, $1 \leq t \leq 2$, and Connection Index CI using formula (1-3)
4) Store the permutation π and the matrix H that give rise to $\max(CI)$, $\min(C_4)$, $\min(C_6)$, $\min(C_8)$. Calculate CI and C_4 for $N_{cw} \times N_{cw}$ interleaver and select the interleaving that meets the requirements.

Next, we present the algorithm for constructing algebraic interleavers and pseudo-random interleavers based on GBIm.

3.1 An interleaver based on Generalized Block Interleaving with Almost Regular Permutation

The Almost Regular Permutation (ARP) interleaver, proposed by Berrou [24, 25] is a widely used algebraic interleaver in transmission systems. It employs a uniform shuffle cycle P combined with a cyclic shift vector S as follows

$$\pi_{ARP}(i) = (P \cdot i + S(i \mod Q + 1)) \mod K + 1, \tag{6}$$

where $i, 1 \leq i \leq K$ is the address of the data bit after interleaving and $\pi_{ARP}(i)$ be the address of the corresponding data bit before interleaving. The natural number P is relatively prime to K, and K is the length of the interleaving, which must be divided by Q, called the Disorder Degree.

We choose prime numbers instead of numbers that are relatively prime with $K = N_{cw}$ as in ARP interleaving to simplify the determination of initial parameters when searching with different interleaving lengths.

Given a vector $S = (s_1, s_2, \ldots, s_n)$ of length n with elements $0 \leq s_j \leq m-1, 1 \leq j \leq n$. The swapping of positions of signal nodes (as well as code nodes) does not change the CI and $C4$ indices. Adding a natural number divided by m to $P \cdot i$ in formula (6) leads to a cyclic shift of the signals selected through the algorithm for finding the optimal interleaving, similar to finding random General Block Interleavers. To simplify the determination of initial parameters when searching with different interleaving lengths, instead of using relatively prime numbers, we choose prime numbers denoted as $P_k, 1 \leq k \leq n$ Specifically, with $i, 1 \leq i \leq K$ the explicit formula for the component interleaver of the General Algebraic ARP [9, 25] interleaving is determined as follows

$$\pi_k(i) = (P_k * i + S_k) \mod N_{cw} + 1. \quad (7)$$

Therefore, the algorithm for generating the generalized algebraic block interleaving is

Algorithm 5 : The algorithm for generating generalized algebraic block interleaving

1) Establish the set P consisting of $n+2$ prime numbers.
2) Establish S:
$$S = \{(s_1, s_2, \ldots, s_n), 0 \leq s_k \leq m-1, 1 \leq k \leq n\} \quad (8)$$
3) For each subset of n prime numbers $\{P_1, P_2, \ldots, P_n\}$ with $P_k \in P, 1 \leq k \leq n$ and each vector S, create the component interleaver $\pi_k(i)$ according to (7).
4) With $1 \leq i \leq N_{cw}$, generate interleaving:
$$\pi_{ARP}\left(i + (k-1)N_{cw}\right) = \left(\pi_k(i) - 1\right) n + k \quad (9)$$
5) Calculate $C4, CI$ according to formula (1-3).

Repeat the steps with all $\frac{(n+2)(n+1)}{2}$ combinations of n prime numbers $\{P_1, P_2, \ldots, P_n\}$ selected from the given $n+2$ prime numbers and all mn vectors S until CI and $C4$ meet the set criteria. The criterion is set as $CI \geq N_s \times 0.9$ and to minimize $C4$. If there are multiple interleavers with the same minimal $C4$, select the interleaving with the highest CI among those interleavers.

3.2 An interleaver based on Generalized Block Interleaving with Golden

Notable pseudo-random interleaving techniques such as the Golden Relative Prime interleaving and the Dithered Relative Prime interleaving have been widely applied in the field of telecommunications [26, 27]. Both methods demonstrate high quality due to the nearly random address relationship between output and input when the interleaving index vectors are determined based on the Golden coefficients $g = \frac{\sqrt{5}-1}{2} \approx 0.618$. However, these algorithms still have several limitations and are not entirely suitable for the BIBCM-ID system structure. In this section, we present the algorithm for constructing the improved Golden interleaving based on the GBIm, in which the component interleaver are constructed based on the algorithm for generating the Golden shuffled vector indices through the Golden Relative Prime and Dithered Golden Relative Prime techniques. The process of determining the shuffle vector indices for the Relative Prime shuffle for component interleaver is performed according to *Algorithm* 6.

Algorithm 6 : Relative Prime interleaving vector index generation algorithm

1) Calculate the Golden index: $g = \frac{\sqrt{5}-1}{2}$
2) Calculate c
$$c = \frac{N_{cw} \cdot (g^m + j)}{r} \quad (10)$$
where m is any nonzero integer, r is the distance between two input bits (a nonzero integer), and j is any integer.
3) Generate the vector e with $k = 1, 2, \ldots, N_{cw}$
$$e(k) = [s + (k \cdot c)] \mod L \quad (11)$$
L is a positive integer greater than 0 used to divide the vector e into different remainder groups, $L = 1, 2, \ldots, N_p$
4) Generate the sorted vector z with $k = 1, 2, \ldots, N_{cw}$
$$a(k) = e(z(k)) \quad (12)$$
where $a = \text{sort}(e)$
5) Compute the interleaving index $\pi_j(z(k))$
$$\pi_j(z(k)) = k \quad (13)$$

For each $j, 1 \leq j \leq n$, execute all steps of *Algorithm* 6 to find n component interleavers. We have the algorithm for constructing the improved Golden interleaving based on the GBIm using component interleavers as follows

Algorithm 7 : Improved Golden interleaving generation algorithm based on the GBIm

1) Calculate interleaving index according to *Algorithm* 6
2) For each i, $1 \leq i \leq N_{cw}$, generate permutation:
$$\pi_{GBI\_Golden}\left(\pi_j(i) + (j-1)N_{cw}\right) = i \cdot n + j \quad (14)$$
3) Calculate $C4, CI$ according to *Algorithm* 1. Calculate CI and $C4$ for $N_{cb} \times N_{cb}$ interleaver and select the interleaving that meets the requirements.

The GBIm demonstrates systematic properties, allowing the construction of interleaver that can be defined through mathematical expressions. Therefore, applying the GBIm interleaving method not only optimizes storage resources but also facilitates flexible system implementation, with interleaving structures that can be easily modified without changing the hardware architecture.

4 Results and Analysic

To evaluate the quality of the BIBCM-ID system using the proposed interleaver, we conducted a simulation of the BIBCM-ID system with the channel encoder being the (8,4,4) Extended Hamming code combined with 16-QAM modulation on an AWGN channel, with a limit of 10 iterations of demodulation/decoding. The interleaver used include block interleaving (BI), random generalized block interleaving (GBI-rand), GBI-ARP and GBI-Golden with interleaving lengths (Np = 960, 1920, 2880, 3840, 5120, 7680, 9240). The evaluation results of the interleaver using Algorithms 1-4 are shown in Table I.

Table I
COMPARISON RESULT OF INTERLEAVERS

Interleavers	N_p	C_4	CI	Elapsed time (s)
BI	960	5040	4	0.065420
	1920	10080	4	0.074479
	2880	15120	4	0.085028
	3840	20160	4	0.107443
	5120	26880	4	0.138985
	7680	40320	4	0.239487
	9240	25410	4	0.376839
Golden	960	1702	38.0833	0.057597
	1920	2775	33.8233	0.070316
	2880	4469	32.9056	0.115437
	3840	5617	32.9083	0.239102
	5120	8060	32.8406	0.435598
	7680	13311	31.0375	1.096680
	9240	18633	34.4524	1.705191
GBI-ARP	960	0	120	2269.637
	1920	0	229.5	5731.964
	2880	0	293.5	19113.996
	3840	0	341	39134.009
	5120	0	383.5	84144.043
	7680	0	415	108987.075
	9240	0	393.2	145472.267
GBI-Golden	960	0	119.23	57.269
	1920	0	215.275	58.206
	2880	0	282.86	605.682
	3840	0	328.17	978.863
	5120	0	369.366	4682.091
	7680	0	410.6854	6936.056
	9240	0	407.9212	9834.053

Analysis of the comparison results of the interleaving parameters, as shown in Table 1, indicates that the proposed GBI interleaver achieves relatively high CI compared to Block interleaver (BI) and basic Golden interleaver, while the number of closed loops ($C4$) is zero. Although both proposed interleaver, the GBI-ARP and the GBI-Golden, meet the criteria for $C4$ and CI indices, the GBI-ARP requires significantly more generation time compared to the improved Golden interleaving using the GBIm. For the generalized algebraic block interleaving, the parameters m and n vary depending on the type of channel code used, and the number of vector S will also change. When m and n have large values, constructing the generalized algebraic block interleaving will consume considerable time to search for and select an appropriate interleaving. In contrast, the generalized Golden block interleaving uses a pseudo-random technique combined with an algebraic function to determine the interleaving index vector for the component interleaver. This method only requires the interleaving length to be known in advance; when the interleaving length needs to be increased, it only requires recalculating the interleaving index vector without changing the number of cases to be considered. This helps reduce complexity and computation time compared to the GBI-ARP. The simulation results of the BIBCM-ID system using popular interleaver and interleaver constructed using the GBIm are shown in Figures 3-4.

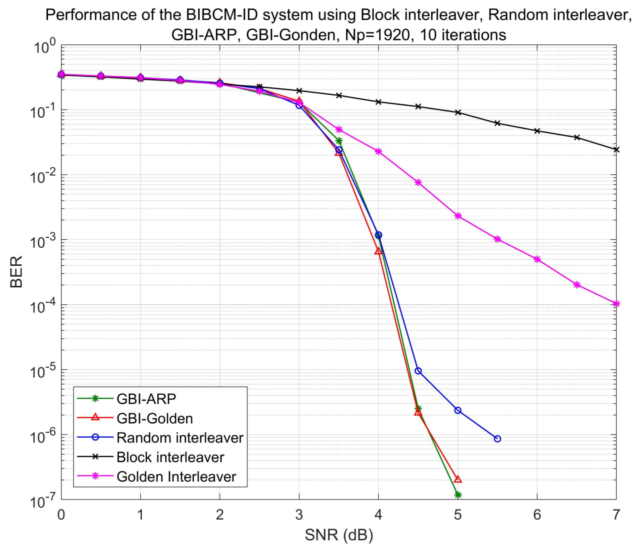

Figure 3. Quality of the BIBCM-ID system using BI, Rand, and GBI interleaving.

Analysis of the simulation results of the BIBCM-ID system, as presented in Figure 3, shows that both interleaver constructed using the GBIm achieve superior performance compared to block interleaving (Block interleaver), Golden Relative Prime interleaving, and significantly lower error floors compared to random interleaving (Random interleaver).

Simulation results show that the GBI-ARP and GBI-Golden interleavers significantly improve the BER performance of the BICM-ID system using (8,4,4) Extended Hamming code and 16-QAM modulation, achieving a gain of 0.5 dB in SNR at a BER of 10^{-6} compared to random interleaver, and over 2dB at a BER of 10^{-4} compared to traditional Block interleaver and basic Golden interleaver. This performance is achieved due to the characteristics of the generalized block interleaving construction method, which allows minimizing the number of short loops while increasing the number of connections between a code bit and signal points and vice versa (Table I). This enhances the exchange of information between code words and signal points, thereby significantly improving extrinsic information between decoding iterations. As a result, the decoding performance is enhanced, and the error floor is improved.

Additionally, the simulation results indicate that selecting an appropriate interleaving length with the

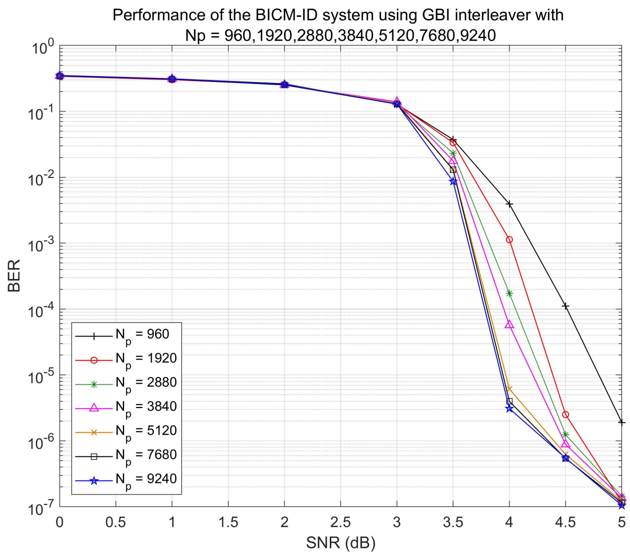

Figure 4. Comparison of the quality of the BIBCM-ID system using algebraic GBI interleaver with different interleaving lengths N_p.

smallest value that still ensures system performance is a crucial factor. Designing interleaver with suitable lengths not only helps create efficient interleaver but also optimizes the use of storage resources of the device and reduces the complexity of data processing. For example, in the case of using the generalized algebraic block interleaving GBI-ARP (Fig. 4), when the interleaving length exceeds 5120 bit-length a certain point, the system quality changes insignificantly.

5 Conclusion

This paper proposes the GBIm method for designing interleavers for the BIBCM-ID system. The GBIm approach introduces a novel design philosophy rooted in the Theory of Relations in Mathematics to model the BICM-ID system. The proposed method showcases systematic characteristics, facilitating the development of interleavers defined through mathematical expressions, along with newly established technical criteria. This framework supports the design, evaluation, and selection of optimal interleavers that enhancing system performance compared to traditional interleaving methods. The GBIm demonstrates systematic properties, allowing the construction of interleaver that can be defined through mathematical expressions. This development increases flexibility for the system. The research findings reveal that the generalized algebraic block interleavings and generalized Golden block interleavers, constructed based on the proposed method, significantly improve the BER performance of the BIBCM-ID system, while also fulfilling criteria for complexity and latency by minimizing the number of iterations during the decoding process, thereby making them suitable for next-generation real-time communication systems.

However, to enhance the comprehensiveness of this study, it is crucial to evaluate the effectiveness of the proposed method across various channel models and conditions, as well as within different system architectures. This research direction will be explored in our future studies to further validate the versatility and efficacy of the proposed method.

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